

## Analyzing Consumer's Behaviour in Risk and Uncertainty Situations

by

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*Abstract.* In the paper we will generalize the Slutsky Equation in risk and uncertainty situations using the compensated and uncompensated demand and some local measures of risk aversion. We will obtain a nonlinear optimization problem of maximizing the expected utility; this problem will be solved using the Kuhn-Tucker method. We use the results to analyze the income and substitution effects of price changes on demand in risk and uncertainty conditions.

*Key words:* compensated demand, risk aversion, Slutsky Equation, uncertainty, uncompensated demand

JEL classification: C61, D11, D53

### 1 Introduction

The risk and uncertainty concepts have been rather recently introduced in the economic field. They were first used in the economic theory in 1944 by von Neumann and Morgenstern, in the paper "Theory of Games and Economic Behaviour". But before that, it was Knight who had inferred the importance of the risk and uncertainty concepts for the economic analysis in the paper "Risk, Uncertainty and Profit" in 1921. A very interesting aspect is that the marginal utility in the context of choices in risky conditions was proposed by Bernoulli in 1738. This is considered the starting point for the neo-classical economy theory.

After the World War II the concept of risk aversion had been studied by Friedman and Savage (1949), and also by Markowitz (1952). Pratt (1964) and Arrow (1965) developed a measure for risk aversion, later improved by Ross (1981). Yaari (1968), Kihlstrom and Mirman (1974) defined the risk aversion in multiple cases. Rothchild and Stiglitz (1970, 1971), Diamond and Stiglitz (1974) studied modalities to measure the risk propensity. Knight was the first to distinguish risk from uncertainty in 1921. He considered that the risk refers to the situations in which the principal could attribute probabilities to random events. The uncertainty refers to the situations in which

one could not attribute and calculate specific probabilities (Keynes).

In 1951, Arrow noted that the most challenging aspect was to specify how risk and uncertainty affect the economic decisions. Therefore, it is very important to establish a connection between the increase or decrease of the uncertainty and the principal's behaviour. Also it is important to know how principals consider the risky situations when the incomes are random. To find the answers, new concepts such as choices in risky situations or in uncertainty needed to be introduced. Hicks (1931) and Marschak (1938) considered that the preferences should be formulated over a probability distribution, but they could not separate the attitude towards risk or uncertainty by the pure preferences over the results. The starting point was random to order randomly the speculations taking into account the mean and the variance.

Arrow (1953) and Debreu (1959) explained the uncertainty using the preferred states. This was used by Hirshleifer (1959, 1966) in the investment theory and further developed by Radner in the financial field and in the general equilibrium theory.

Nowadays, the risk and uncertainty concepts are largely used in theoretical and empirical studies analysing economic agents' behaviour, the literature on these topics being really huge.

In the first part of the paper we will present some classical concepts from microeconomic theory regarding optimal choices of the consumer: uncompensated and compensated demand, the indirect utility function and the expenditure function. These concepts are used for deriving one of the most important results of consumer's theory – the Slutsky Equation. This result shows the relationship between price derivatives of Hicksian and Walrasian demands, relation that helps to analyze the income and substitution effects (the effect of price changes on demand). It is also helpful in summarizing the goods properties: if they are substitutes or complements. In the second part of the paper we will consider an investor (a consumer) with an initial investment capital (initial endowment) he can invest. We will suppose there are two investment opportunities: one part of the initial capital is invested in one risky active and the other part is invested in a safe (without risk) active. The randomness' return of the risky asset generates a new optimization problem for maximizing the consumer's expected utility. We will solve this problem using some local measures of risk aversion (the relative and absolute risk aversion index, risk premium and the certainty equivalent), to analyze the income and substitution effects– in risk and uncertainty conditions and we will derive a generalized Slutsky Equation.

## 2 The Classical Demand Theory

In the classical approach to consumer's demand, the analysis of consumer behaviour begins by specifying the consumer's preferences over the commodities bundles in the consumption set,  $X \subset R_+^n$ . The consumer's preferences are represented by a preference relation  $\succeq$  defined on  $X$ , relation that is considered to be rational, complete, reflexive and transitive. Other types of assumptions regarding this relation are needed for modelling consumer's behaviour: continuity, monotonicity (or local non satiation) and convexity.

Alternatively – and this is the way we adopt in this section for analytical purposes – we can summarize the consumer's preferences by

means of a utility function. We assume throughout this section that the consumer has preferences represented by a continuous utility function  $U : X \rightarrow R$ , where  $X$  is the consumption set, containing all possible commodities bundles, whose prices are denoted by  $(p_1, p_2, \dots, p_n)$  or by the price vector  $p \gg 0$ .

### *Uncompensated demand functions*

We now turn to the study of the consumer's decision problem, which is called *utility maximization problem* (Mas-Colell et al, 1995). A rational consumer will always choose a consumption bundle (the most preferred) from the set  $X$ , given the prices  $p \gg 0$  and the wealth level (income level)  $w > 0$ , in order to maximize her utility level. So, the problem of utility maximization can be stated as:

$$(P) \quad \begin{aligned} & \max_{x \geq 0} U(x) \\ & s.t. \\ & px \leq w \end{aligned}$$

We denote by  $X(p, w)$  the optimal (vector) solution of this problem, which is called *the vector of Walrasian (or uncompensated) demand functions* and has the following properties (Mas-Colell et al, 1995):

- i) is homogeneous of degree zero in  $(p, w)$ ;
- ii) satisfies Walras law:  $px = w$ ;
- iii) if  $U(\cdot)$  is strictly concave, the solution of (P) is unique.

The optimal value of the objective function in problem (P) is denoted by  $V(p, w)$ . The function  $V(p, w)$  is called *the indirect utility function* and has the properties (Mas-Colell et al, 1995, Varian, 1984) listed below:

- i)  $V(p, w)$  is continuous for all  $p \gg 0, w > 0$ ;
- ii)  $V(p, w)$  is nonincreasing in  $p$  and nondecreasing in  $w$ ;
- iii)  $V(p, w)$  is quasi-convex in  $p$ ;
- iv)  $V(p, w)$  is homogeneous of degree zero in  $(p, w)$ .

### *Compensated demand functions*

Instead of searching for the maximal level of utility that can be obtained given a wealth  $w$ , we can find the minimal amount (level) of wealth required to reach a given utility level  $u$ . This

analysis consists in deriving and solving a “dual” problem to (P), namely *the expenditure minimization problem*:

$$(D) \quad \begin{aligned} & \min_{x \geq 0} px \\ & s.t. \\ & U(x) \geq u \end{aligned}$$

The optimal solution of this problem is called *the vector of Hicksian (or compensated) demand functions* and represents the least costly bundle that allows the consumer to obtain the utility level  $u$ , given the prices  $p \gg 0$ . We denote by  $\varphi(p, u)$  this solution. It satisfies the following properties (Mas-Colell et al, 1995):

- i) is homogeneous of degree zero in  $(p, w)$ ;
- ii) satisfies Walras law:  $px = w$ ;
- iii) if  $U(\cdot)$  is strictly concave, the solution of (P) is unique.

The optimal value of the objective function in problem (D) is denoted by  $C(p, u)$  and it is called *the expenditure function*. Its properties are summarized below (Mas-Colell et al, 1995, Varian, 1984):

- i)  $C(p, u)$  is nondecreasing in  $p$ ;
- ii)  $C(p, u)$  is concave in  $p$ ;
- iii)  $C(p, u)$  is homogeneous of degree one in  $p$ ,  $p \gg 0$ ;
- iv)  $C(p, u)$  satisfies Shepard’s Lemma:

Assuming that the derivative exists and for

$$p \gg 0, \text{ then } \frac{\partial C(p, u)}{\partial p_i} = \varphi_i(p, u), \text{ for } i = 1, 2, \dots, n.$$

*Relationships between Demand functions, Indirect Utility function and Expenditure Function – Slutsky Equation* (Varian, 1984)

There are several important identities – relationships between demand functions, indirect utility function and expenditure function. Let us consider the two problems (P) and (D), with their respective solutions. We can now list the following properties:

- 1) The minimal expenditure needed to obtain the utility  $u = V(p, w)$  is  $w$ :

$$C(p, V(p, w)) = w.$$

- 2) The maximal utility obtained with an income equal to  $C(p, u)$  is  $u$ :

$$V(p, C(p, u)) = u.$$

- 3) The uncompensated demand at the income  $w$  is the same as the compensated demand at utility level  $V(p, w)$ :

$$X_i(p, w) \equiv \varphi_i(p, V(p, w)), \quad i = 1, 2, \dots, n.$$

- 4) The compensated demand at the utility level  $u$  is the same as the uncompensated demand at the income  $C(p, u)$ :

$$\varphi_i(p, u) \equiv X_i(p, C(p, u)), \quad i = 1, 2, \dots, n.$$

We can state now the following proposition, representing the Slutsky Equation.

**Proposition (The Slutsky Equation)** (Varian, 1984): Suppose that  $U(\cdot)$  is a continuous utility function representing a preference relation defined on the consumption set. Then, for all  $(p, w)$  and  $u = V(p, w)$  we have:

$$\frac{\partial X_j(p, w)}{\partial p_i} = \frac{\partial \varphi_j(p, u)}{\partial p_i} - \frac{\partial X_j(p, w)}{\partial w} \cdot X_i(p, w),$$

for all  $i, j$ .

*Proof*

We consider that the consumer facing the price vector  $\bar{p} \gg 0$  and having an income equal to  $\bar{w}$  obtain a utility level  $\bar{u}$ , so that  $V(\bar{p}, \bar{w}) = \bar{u}$ . We note that the income  $\bar{w}$  must satisfy the first identity,  $C(\bar{p}, \bar{u}) = \bar{w}$ . The solutions (uncompensated and compensated demands) of the two optimization problems (P) and (D) satisfy the identity:

$$\varphi_j(p, u) \equiv X_j(p, C(p, u)), \text{ for all } j = 1, 2, \dots, n$$

We differentiate this relation with respect to  $p_i$  and we evaluate it in  $(\bar{p}, \bar{u})$ . Therefore, we get:

$$\frac{\partial \varphi_j(\bar{p}, \bar{u})}{\partial p_i} = \frac{\partial X_j(\bar{p}, C(\bar{p}, \bar{u}))}{\partial p_i} + \frac{\partial X_j(\bar{p}, C(\bar{p}, \bar{u}))}{\partial w} \cdot \frac{\partial C(\bar{p}, \bar{u})}{\partial p_i}$$

But, from Shepard Lemma, we have:

$$\frac{\partial C(p, u)}{\partial p_i} = \varphi_i(p, u)$$

so this can be used in the previous relation:

$$\frac{\partial \varphi_j(\bar{p}, \bar{u})}{\partial p_i} = \frac{\partial X_j(\bar{p}, C(\bar{p}, \bar{u}))}{\partial p_i} + \frac{\partial X_j(\bar{p}, C(\bar{p}, \bar{u}))}{\partial w} \cdot \varphi_i(\bar{p}, \bar{u})$$

We finally use the identities written for the values  $(\bar{p}, \bar{u})$ , i.e.  $C(\bar{p}, \bar{u}) = \bar{w}$  and  $\varphi_i(\bar{p}, \bar{u}) = X_i(\bar{p}, C(\bar{p}, \bar{u})) = X_i(\bar{p}, \bar{w})$  and rearranging the terms we get:

$$\frac{\partial X_j(\bar{p}, \bar{w})}{\partial p_i} = \frac{\partial \varphi_j(\bar{p}, \bar{u})}{\partial p_i} - \frac{\partial X_j(\bar{p}, \bar{w})}{\partial w} \cdot X_i(\bar{p}, \bar{w})$$

The Slutsky Equation shows a decomposition of the effect of a price change on quantity demanded in two different effects:

- a *substitution effect*, which is defined as the effect of a price change on a quantity demanded due exclusively to the fact that its relative price has changed;

- an *income effect*, which is defined as the effect of a price change on a quantity demanded due exclusively to the fact that the consumer's real income has changed.

The left hand side of the above relation

$$\left( \frac{\partial X_j(\bar{p}, \bar{w})}{\partial p_i} \right)$$
 represents the change in

uncompensated demand holding expenditure fixed at  $\bar{w}$  when  $p_i$  changes (change in usual demand). The right hand side is a sum of two

terms: the first term  $\frac{\partial \varphi_j(\bar{p}, \bar{u})}{\partial p_i}$  shows how

compensated demand changes when  $p_i$  changes – the substitution effect and the last one, including the sign,

$$\left( \frac{\partial \varphi_j(\bar{p}, \bar{u})}{\partial p_i} - \frac{\partial X_j(\bar{p}, \bar{w})}{\partial w} \cdot X_i(\bar{p}, \bar{w}) \right)$$
 is equal to the

change in demand when income changes multiplied by the corresponding uncompensated demand (the change in income to keep utility constant).

### 3 A Generalization of Slutsky Equation in risk and uncertainty conditions

In the classical settings, different authors studied optimal consumer's choices that result in perfectly outcomes. In reality, many important economic decisions involve some elements of risk. In this section we focus on the special case in which the outcome of a risky choice is an amount of money (Marinescu et al, 2008).

We consider an economic agent representing the private investors that has an initial income  $w_0$  (the initial endowment). He has two choices (or investment opportunities): to invest in one safe active (without risk), with a rate of return denoted by  $r$  or to invest in a risky active whose

rate of return is a random variable  $\tilde{e}$  with mean and variance finite.

We denote by  $a$  the proportion of initial endowment invested in risky active.

The agent's income at the end of the first period will be:

i) The agent invests  $aw_0$  in risky active and at the end of the first period he will have

$$aw_0(1 + e) \text{ (for a certain value } e \text{ for } \tilde{e} \text{).}$$

ii) The agent invests  $(1-a)w_0$  in active without risk and he will have at the end of the first period  $(1-a)w_0(1+r)$ .

So, at the end of the first period he will obtain:

$$w(e) = w_0a(1 + e) + w_0(1-a)(1+r)$$

or

$$w(e) = w_0[1 + ae + (1-a)r]$$

We consider that the agent is risk adverse and let  $U(\cdot)$  be the von Neumann-Morgenstern utility function with the usual properties:  $U'(\cdot) > 0$ ,  $U''(\cdot) < 0$ .

Let  $\bar{e}$  be the mean value of  $\tilde{e}$  and we denote by  $\sigma^2$  its variance, negligible with respect to the higher moments. We consider the following notations:

$$E[\tilde{x}_1] = E\{aw_0(1 + \tilde{e})\} = \bar{x}_1 \text{ and } \sigma_{\tilde{x}_1}^2 = (a^2w_0^2)\sigma^2$$

We will use an alternative way of analyzing consumer's choices and we consider the utility function  $B(\bar{x}_1, x_2) = EU(\tilde{x}_1 + x_2)$ , approximated by a dependent function on  $\bar{x}_1$  and  $x_2$ . For this, we will use the concept of *certainty equivalent*, denoted by  $E_C(\bar{x}_1 + x_2)$ .

The certainty equivalent satisfies the equation:

$$B(\bar{x}_1, x_2) = EU(\tilde{x}_1 + x_2) = U(\bar{x}_1 + x_2 - E_C(\bar{x}_1 + x_2)) \quad (1)$$

We will also consider the following notations:  $z = \tilde{e} - \bar{e}$ ,  $\sigma_z = \sigma$ . Hence  $E(z) = 0$  holds. Then:

$$B(\bar{x}_1, x_2) = E[U(\bar{x}_1(1+z) + x_2)] \quad (2)$$

We will approximate the function from (2) by a series expansion:

$$U(\bar{x}_1 + x_2 + \bar{x}_1 z) \approx U(\bar{x}_1 + x_2) + U'(\bar{x}_1 + x_2) \cdot \frac{\bar{x}_1 z}{1!} + \frac{U''(\bar{x}_1 + x_2)}{2!} \cdot \bar{x}_1^2 z^2 \quad (3)$$

The right side of (1),  $U(\bar{x}_1 + x_2 - E_C(\bar{x}_1 + x_2))$ , could be approximated (from Lagrange's Theorem) by:

$$U(\bar{x}_1 + x_2 - E_C(\bar{x}_1 + x_2)) = U(\bar{x}_1 + x_2) - U'(\bar{x}_1 + x_2)E_C(\bar{x}_1 + x_2) \quad (4)$$

From (3), taking the "expected value" of both sides (applying the operator "E") we will get:

$$EU(\tilde{x}_1 + x_2) = U(\bar{x}_1 + x_2) + 0 + \frac{U''(\bar{x}_1 + x_2)}{2!} \bar{x}_1^2 \sigma^2 \quad (5)$$

From (4) and (5) we will obtain:

$$E_C(\bar{x}_1 + x_2) = -\frac{U''(\bar{x}_1 + x_2)}{U'(\bar{x}_1 + x_2)} \frac{\bar{x}_1^2 \sigma^2}{2} = \frac{\bar{x}_1^2 \sigma^2}{2} r_a(\bar{x}_1 + x_2)$$

where  $r_a(\cdot)$  represent the absolute index of risk aversion.

We will approximate  $B(\tilde{x}_1, x_2)$  by:

$$B(\bar{x}_1, x_2) = U\left(\bar{x}_1 + x_2 - \frac{\bar{x}_1^2 \sigma^2}{2} r_a(\bar{x}_1 + x_2)\right) \quad (6)$$

From the definition of  $\bar{x}_1$  and  $x_2$ , we will get:

$$w_0 = \frac{\bar{x}_1}{1+e} + \frac{x_2}{1+r}$$

$$\text{or } \bar{x}_1 = w_0(1+e) - \frac{1+e}{1+r} x_2 \quad (7)$$

The equation (7) is the income equation or welfare equation.

We define  $p_e = \frac{1}{1+e}$  and  $p_r = \frac{1}{1+r}$ , then the

above equation could be written as:

$$w_0 = p_e \bar{x}_1 + p_r x_2.$$

Therefore, the problem of choosing the optimal portfolio is:

$$\begin{aligned} & \max_{\bar{x}_1, x_2} \tilde{B}(\tilde{x}_1, x_2) \\ & \text{s.t. } p_e \bar{x}_1 + p_r x_2 = w_0 \end{aligned}$$

or

$$\begin{aligned} & \max_{\bar{x}_1, x_2} U\left(\bar{x}_1 + x_2 - \frac{\sigma^2 \bar{x}_1^2}{2} r_a(\bar{x}_1 + x_2)\right) \\ & \text{s.t. } p_e \bar{x}_1 + p_r x_2 = w_0 \end{aligned} \quad (8)$$

From Gossen's Law, the optimal solution must satisfy the first order condition:

$$\frac{\frac{\partial U}{\partial \bar{x}_1}}{\frac{\partial U}{\partial x_2}} = \frac{1+r}{1+e} = \frac{p_e}{p_r} \quad (9)$$

The welfare equation and the condition (9) form a system of equations. Solving this system we will obtain the optimal solutions  $\bar{x}_1^*$  and  $x_2^*$ .

Let  $X(p_e, p_r, w_0)$  be the uncompensated demand and  $V(p_e, p_r, w_0)$  be the indirect utility function, both having the usual properties.

Consider a fixed utility level for the economic agent, denoted by  $b$ .

The goal of our analysis is to determine the minimal initial endowment that consumer needs to attain the utility level  $b$  (i.e. the utility level associated to one particular social category or for people living in a community).

Now we can state the following nonlinear optimization problem:

$$\begin{aligned} & \min_{\bar{x}_1, x_2} [p_e \bar{x}_1 + p_r x_2] \\ & \text{s.t. } \tilde{B}(\bar{x}_1, x_2) = b \end{aligned} \quad (10)$$

The solution of this problem is  $(\bar{\varphi}_1, \varphi_2)$  or the

vector  $\Phi(p_e, p_r, b) = \begin{pmatrix} \bar{\varphi}_1 \\ \varphi_2 \end{pmatrix}$ . Then, the minimal

expenditure function is  $C(p_e, p_r, b) = p_e \bar{\varphi}_1 + p_r \varphi_2$  and it has the following properties: it is concave and increasing in prices and  $b$  and satisfies the Shephard's Lemma.

Differentiating the relationships between the functions  $X(\cdot)$  and  $\Phi(\cdot)$ , we will get:

$$\frac{\partial \Phi_{\bar{x}_1}}{\partial p_e} = \frac{\partial X_{\bar{x}_1}(p_e, p_r, w)}{\partial p_e} + \frac{\partial X_{\bar{x}_1}(p_e, p_r, w)}{\partial w} \cdot \frac{\partial w}{\partial p_e} \quad (11)$$

or

$$\frac{\partial X_{\bar{x}_1}(p_e, p_r, w)}{\partial p_e} = \frac{\partial \Phi_{\bar{x}_1}}{\partial p_e} - \frac{\partial X_{\bar{x}_1}(p_e, p_r, w)}{\partial w} \cdot \bar{x}_1^* \quad (12)$$

The equation (12) shows how the price effect (the left side of equation) can be decomposed into a substitution effect and an income effect (the first and respectively, the second term of the right side).

Analogous, it follows:

$$\frac{\partial X_{\bar{x}_1}(p_e, p_r, w)}{\partial p_r} = \frac{\partial \Phi_{\bar{x}_1}}{\partial p_r} - x_2^* \frac{\partial X_{\bar{x}_1}(p_e, p_r, w)}{\partial w} \quad (13)$$

$$\frac{\partial X_{x_2}(p_e, p_r, w)}{\partial p_r} = \frac{\partial \Phi_{x_2}}{\partial p_r} - x_2^* \frac{\partial X_{x_2}(p_e, p_r, w)}{\partial w} \quad (14)$$

$$\frac{\partial X_{x_2}(p_e, p_r, w)}{\partial p_e} = \frac{\partial \Phi_{x_2}}{\partial p_e} - \frac{\partial X_{\bar{x}_1}(p_e, p_r, w)}{\partial w} \cdot \bar{x}_1^* \quad (15)$$



The equations (12), (13), (14) and (15) are combined into the matrix form of Slutsky's Equation:

$$\nabla_p X(p, w) = \nabla_p \Phi(p, b) - \nabla_w X(p, w) \cdot (\bar{x}_1^*, x_2^*) \quad (16)$$

#### 4 Conclusions

As we already saw, in its standard form the Slutsky Equation decomposes the change in demand due to a price change into a substitution effect and an income effect and the income effect was due to a change in the purchasing power when prices changed. There was a strong assumption made in that model: the money income was held constant. In a revisited version of the Slutsky Equation, Varian examines the case where the money income changes since the value of the consumer's endowment changes when the prices change; he then shows that the income effect can be decomposed into an ordinary income effect (when the price changes, the purchasing power also changes) and an endowment income effect (a price change also affects the consumer's endowment bundle so that the money income changes) (Varian, 2002). The Slutsky Equation was also used as a standard microeconomic tool to analyse the change in demand due to an interest rate change into income effects and substitution effects (consumer's intertemporal choices).

Our approach was based on some classical microeconomic concepts from consumer's theory, but we analysed the case where the consumer's choices are made in risk and uncertainty conditions, such that we were able to derive a generalized form of the Slutsky Equation. The model we proposed here could be adapted for analyzing the evolution of physical or chemical processes (or any type of process), having a risk when producing. Hence, the goal

of the model is to minimize the costs to obtain a certain result, a priori fixed.

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#### References

- Cass, D. and Stiglitz, J.E. (1972), Risk Aversion and Wealth Effects on Portfolios with Many Assets, *Review of Economic Studies*, 39(3):331-354
- Fishburn, P.C. and Burr Porter, R. (1976), Optimal Portfolios with One Safe and One Risky Asset: Effects of Changes in Rate of Return and Risk, *Management Science*, 22(10):1064-1073
- Laffont, J.J. (1999) *Economie de l'Incertain et de l'Information*, Economica, Paris
- Mas-Colell, A., Whinston, M.D. and Green, J.R. (1995), *Microeconomic Theory*, Oxford University Press
- Marinescu, D. and Marin, D. (2009), Risk and Uncertainty: Analyzing the Income and Substitution Effects, *Economic Computation and Economic Cybernetics Studies and Research*, 1: 41-54
- Marinescu, D., Ramniceanu, I. and Marin, D. (2008), The Influence of the Income Taxation on the Agent Savings, *Symposium on Computer Applications in Science, Engineering and Education*, CASEE 2008, Crete, Greece
- Varian, H.R. (1984), *Microeconomic Analysis*, Second Edition, Norton and Co, New York
- Varian, H.R. (2002), *Intermediate Microeconomics. A modern approach*, Sixth Edition, Norton and Co, New York and London

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